MEMBRANE STRUCTURES ASSOCIATION OF AUSTRALASIA
Sydney Science Centre 35 Clarence St, Sydney. 2000. Telephone (02) 29 7747

Peter Kneen

Senior Lecturer
Department of Structural Engineering
University of New South Wales

COMPUTER GENERATION OF SHAPES AND CUTTING PATTERNS

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Peter Kneen
Senior Lecturer
Department of Structural Engineering
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1. INTRODUCTION

This paper is primarily concerned with computer simulation of the shape of tension fabric structures together with the associated problem of determining suitable fabric cutting patterns to convert the shape into reality.

Tension fabric structures typically are more complex in their geometric form than are the air-supported forms of fabric structures. They are characterised by an anticlastic surface with a variety of internal and boundary support conditions. Perimeter boundaries can be fixed in shape and position such as beams and arches or can be free such as an unrestrained edge cable attached to isolated anchorages. Internal supports are often required in order to give the structure sufficient curvature which in turn limits the stress levels in the fabric to acceptable values. The internal supports can be of a point kind or a line type, examples being masts (with or without suspended ring beams), alternating portal frames and internal arches.

The shape simulation of the fabric surface is desirable to all parties involved in a project. It will assist the designer to determine clearances, drainage, architectural impact and enable a structural analysis to be attempted. The client will require an appreciation of the shape of the finished product and its visual qualities. A fabricator will use the shape to determine the required cutting patterns.

Traditionally, physical models were used for all aspects. The design process was quite time consuming and often involved building a sequence of more complex and exact models with the final model being used for the patterning. Changes to the positions of supports or types of supports generally cannot be easily incorporated in the more sophisticated models.

It is felt that there is still a role for physical models but this is restricted to three types of models. Firstly small scale architectural models for the principal benefit of the client. Measurements taken from such models would not be accurate enough for use in the detailed design phase. A second type of model which may be constructed for large and important projects is a model that can be instrumented and used in a wind tunnel to obtain wind load data. It should be pointed out that very few projects can afford the cost or time for such testing. The final type of model is termed a "proving model". In this case, the calculated cutting patterns are used to fabricate a scaled down version of the final structure including such details as edge cable sleeves, cables, turnbuckles and other intended means of adjustment. It would be expected that this model would accurately portray the characteristics of the real structure. Construction of this model is time consuming and engrossing.

This leaves a considerable gap in the design process from a conceptual shape, expressed as rough drawings or a crude small model, to the final cutting patterns. This gap is now being successfully filled by computer models of the surface.

2. COMPUTER SHAPE GENERATION

Because of the freeform nature of tensioned fabric structures there are very few structures built which could be accurately defined using an explicit mathematical equation. An exception could be a hypar type structure. It therefore becomes necessary for a numerical solution to be adopted. For the computer, it normally suffices to remember the (x,y,z) coordinates of a finite number of points on the surface. Several hundred such points may be required depending on the degree of symmetry and the extent of the structure.

Several techniques are available to establish these points. Obviously the designer can directly input the coordinates but this defeats the purpose of the computer to a large extent. Alternatively, a small number of controlling positions may be given explicitly and a sequence of intermediate points calculated according to one of many numerical procedures. Finally the points can be digitized by measuring either a model or sketch drawings. Generally speaking the second method is adopted and is described in more detail in the next section.

There is no guarantee that the points so positioned will be the correct ones for the fabric surface. The correct positions should take into account that the fabric is made from a real material and is prestressed by substantial forces. All parts of the surface must be in equilibrium under the action of the loads. For shape definition, the loads are due to the prestress and the membrane self weight which is often ignored as it is small in comparison.

For a carefully constructed physical model, perhaps using a thin rubber sheet or nylon stocking material, a close approximation to the final "correct" shape is automatically achieved when the membrane material is stretched to the supports and boundaries. Another form of physical model involves using soap films which attach to other elements representing edge cables, masts, arches etc. The soap film has equal surface tensions in all directions and this criteria serves as a good starting point for many tension and air supported fabric structures.

Fortunately there is a mathematical procedure available in which the behaviour of a soap film may be simulated by the computer. The procedure is known as the finite element method in which the fabric surface is broken up into a finite number of smaller connected patches called elements. Each element is generally triangular or quadrilateral. By combining sufficient of these elements together, the complete surface may be modelled including curved boundaries, masts, beams and arch type supports.

The method has developed equations relating the forces acting at the corners of the elements to the movements of the nodes. This involves describing the material characteristics. For adjoining elements the nodal forces are combined into a system of equations which will relate the applied loads at all the node positions to the corresponding movements of the nodes.

Nodal coordinates are adjusted to satisfy equilibrium requirements (sum of all forces at a point in any direction is zero). The adjustment process is done in numerous stages and the nodal positions will converge to the "correct" locations that would be adopted either by a soap film or the real membrane material. The details of the process are reasonably complex and are beyond the scope of this paper.

The method requires a sensible layout of these smaller elements. Generally more elements are used in regions where curvatures are highest or the boundaries more curved. If fabric stresses are thought to be changing more rapidly across a region then more elements can be positioned in those locations.

2.1 Initial Layout of Nodes

The finite element method provides one means of transforming an initial shape into a more correct or refined shape by the simulation of soap film elements. The problem remains of establishing the initial shape. One method consists of breaking the surface up into three or four sided regions or surface patches. Such patches may extend right across the structure terminating at known or assumed boundary locations. If positions of the corners of the patches are known then numerical procedures based on fitting curves to these points can be used.

Depending on the amount of information available the surface can be modelled accordingly. Thus if only three points are known in position then a triangular plane is possible whereas if four corners are defined then a doubly curved hypar region is formed. More complex surfaces are available if the positions of midside nodes in the patch are also known as shown in Figure 1.

The coordinates of a point on the surface within the patch (x,y,z) are calculated in terms of the control coordinates at the corners by combining with suitable blending functions, thus

$$x = \sum_{i=1,n} B_i \; X_i \qquad y = \sum_{i=1,n} B_i \; Y_i \qquad z = \sum_{i=1,n} B_i \; Z_i \qquad (1)$$

where n is the number of control points in the patch

B_i is the blending function for control point i

X_i, Y_i, Z_i, are the coordinates of control point i

Each patch has a local set of axes which generally range from -1 to +1 in value across the patch. These may be denoted by the local s and t axes. The blending functions B_i then depend on the complexity of the patch. For a simple quadrilateral region as in Figure 2 the x coordinate of a point may be found from

$$x = B_1 \; X_1 + B_2 \; X_2 + B_3 \; X_3 + B_4 \; X_4 \qquad (2)$$

in which

$$\begin{aligned} B_1 &= (1-s)(1-t)/4 \qquad (3) \\ B_2 &= (1+s)(1-t)/4 \\ B_3 &= (1+s)(1+t)/4 \\ B_4 &= (1-s)(1+t)/4 \end{aligned}$$

Thus when $s = t = 0$, $B_1 = B_2 = B_3 = B_4 = \frac{1}{4}$ and x is then the average of the x coordinates of the four corners. At any corner, say $s = -1$, $t = -1$ then $B_1 = 1$ and $B_2 = B_3 = B_4 = 0$ thus ensuring that $x = X_1$ or that the surface passes through the point. Similar equations apply to the y and z coordinates.

To obtain a layout of points it suffices to calculate the coordinates within a patch or region by dividing the region up into equal intervals in the local s and t axes system.

If there is more information available about the control points of a patch, such as the slopes of the surface in each direction at the corners, then it is possible to ensure that adjacent regions will match up with continuity of slopes.

More complex forms of the blending functions are used than those indicated in equation (3) but the general process as described by equation (1) remains unaltered. The design of many objects is now done using these methods including motor vehicles.

2.2 Generation of Conical Surfaces

Several conical type tension fabric structures have been built in Australia in the past few years. Two of these, designed by the author, are the shade structures in the Queen Street Mall, Brisbane and the amphitheatre structure at Euroa, Victoria. In both cases the fabric was attached to a tensioning ring located around the central mast. Catenary edge cables formed the perimeter. See Figures 3 and 4.

Generation of the initial layout of nodes for both structures was done in the same fashion. Firstly, for each edge cable, the two end point positions and a mid point was specified to enable a parabolic curve to be determined. Each edge curve was divided into an integral number of segments. A curve was formed from each of these edge points to a corresponding point in the central ring. The point on the tension ring was such as to be in the plane of the mast and the point on the edge cable. The shape of this radial curve was parabolic which required defining the slope at the tension ring. A sequence of such curves were used as shown in Figure 5.

A desktop computer (HP-85) with an attached plotter was used for both structures. The initial shapes generated were plotted to scale from different viewing positions, and, after a few trials at locating the mid points of the edge cables and their slopes at the tension ring, the initial shapes were accepted as being the "correct" or refined shapes.

3. COMPUTER PACKAGES

Since these structures were constructed the computer programs have been rewritten for mini and miniframe computers using Fortran. Two spearate programs are now used. FABDES (FABric structures DESign) is used for generating initial shapes, for interactive graphics and for producing cutting patterns. FABDES also generates the input to the second program LISA developed by E. Haug for the analysis of lightweight structures. LISA is based on the finite element method and uses an iterative procedure for the non linear analysis. The soap film analogy is incorporated as part of LISA.

Two recent structures were designed in part using both programs. A complex conical structure of some 970 m^2 with little symmetry was generated initially as described above and then modelled using the soap film feature. Data was passed from FABDES to LISA by means of permanent data files on disk storage. Following the shape refinement process, another set of files were produced by LISA which subsequently were accessed by FABDES to calculate the cutting patterns.

The NSW Mobile Stage erected in the Domain for the Festival of Sydney 83-84 was also a combined effort of using some input from a physical model to define the positions of midpoints of edge cables and tops of the various masts. The three different regions on each side of the centreline were generated separately.

The soap film computer model gave the shape between the major cables. Because each region was done separatley, the dividing ridge cables were fixed to the position indicated by measurement from the model.

It proved rather difficult to match up both the computer model to the fine details of the physical model and some design modifications were necessary.

In both these structures the cutting patterns were plotted out to a 1:25 scale by the computer. A fabric model was constructed directly from the cutting patterns representing each strip and seam in the final structure. In each case a wrinkle free model resulted thus "proving" the cutting patterns.

4. COMPUTER GENERATION OF CUTTING PATTERNS

The preceding sections have outlined how the computer can be programmed to set out a mesh of nodal positions representing an initial approximation to the desired surface shape. Optionally these nodes may be moved as a result of modelling a soap film between known boundary locations such as anchorages and masts. The refined positions of the nodes, stored as sets of x,y, z coordinates, are then used as the basis for the calculation of the cutting patterns.

Unfortunately it is unlikely that the locations of the nodes will correspond to the edges of the rolls of fabric. It is very difficult to predict where the edges would occur on the three dimensional surface due to the varying curvatures in both orthogonal directions.

Considerations of major stress directions and the need to reduce wastage of material leads to adopting different patterns for different regions of the structure. The patterns used for the Queen Street Mall shade structures are shown in Figure 6 together with the location of the generated node positions.

Means of defining the plan position of the two sides of the strip as well as the curvatures at the start and end of the strips are incorporated into FABDES. Essentially the four corner points are described. Each side is considered to be parabolic in plan view. The two parabolas are defined in terms of a central "bow" to the left or right of a straight edge. A strip which is straight in plan will generally be bowed when the 3D surface is flattened out giving an inefficient usage of fabric. Specifying counteracting bows in plan view can lead to near optimal use of material. (See Figure 7).

Having defined the plan outline of a strip, each side is divided into a given number of divisions. From the (x, y) coordinates of points along each side the z coordinate on the surface is calculated from a least squares fit of neighbouring nodal points. By triangulating the strip and using the true lengths of each side of the strip triangles in turn, the strip can be flattened out and orientated along a roll of fabric.

For fabrication purposes it is desirable to provide offsets from a reference line to each side of the strip at regular intervals of say 500 mm. The reference line could be one edge of the roll or along the centreline. A standard Lagrangian or Spline interpolation scheme can be used for this purpose from the flattened out points.

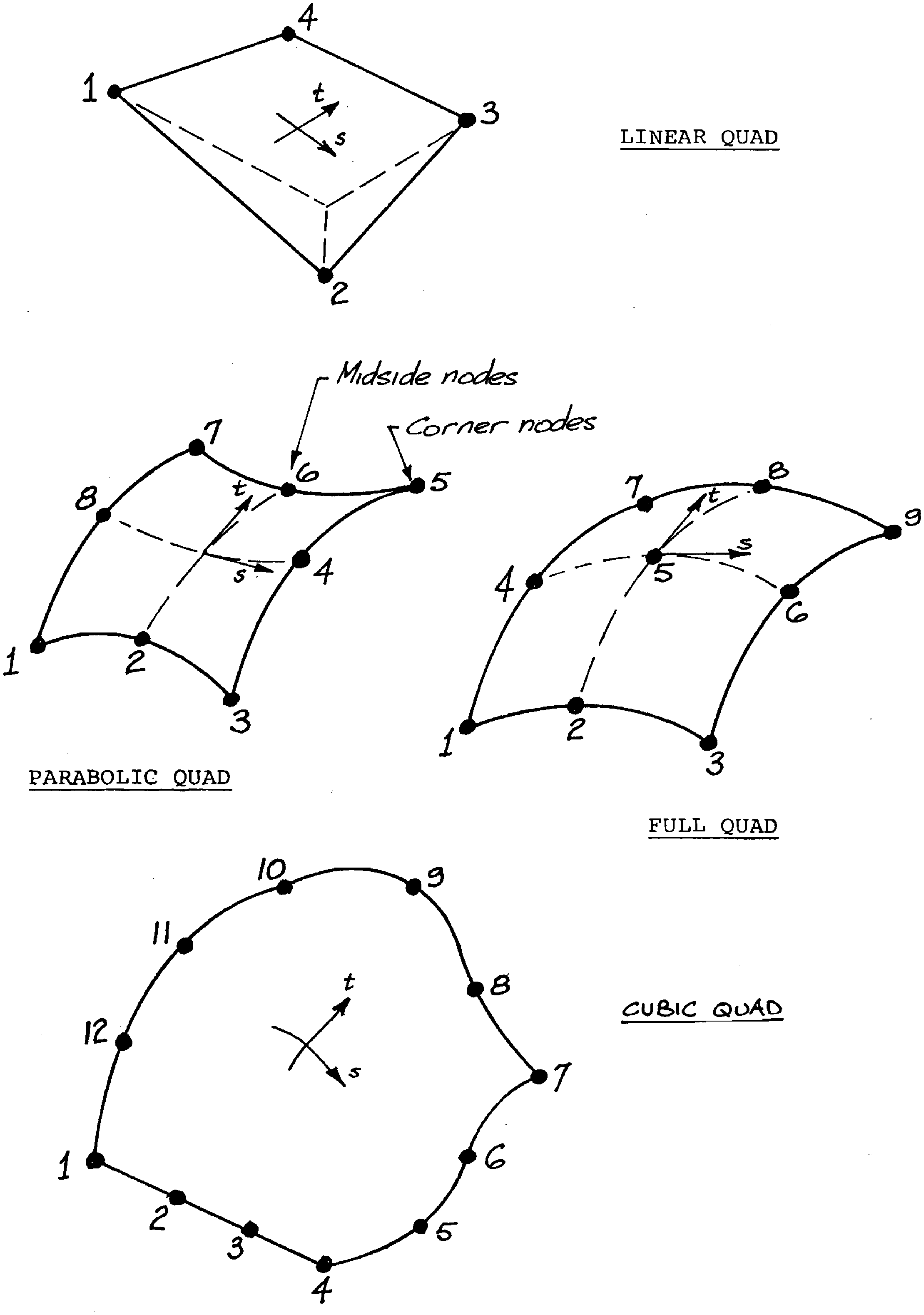
Since the final structure is prestressed and the fabric on the roll is unstressed, then suitable compensation factors can be used to allow for the later elongations. In FABDES this is done by specifying compensation factors in width (weft) at the start, middle and ends of the strip. A length compensation (warp) is also specified. Values of these factors depend on the material being used and may be in the region of zero to 2 percent.

FABDES allows the user to design each strip individually in an interactive fashion. Checks are carried out to compare the maximum width, allowing for the seams, with the width of the rolls. Normally after a strip is acceptable an adjacent strip is considered. FABDES is able to ensure that the new strip will automatically have the same plan layout for the common edge.

After all strips are completed, optional hard copy plotted output is available which can then be given to the fabricator. Several check dimensions are given, such as diagonal distances between corners and total edge lengths. These should detect whether, in marking out, if one set of offsets was overlooked.

5. CONCLUDING REMARKS

The paper has briefly described the computer generation of shapes and cutting patterns for fabric structures. Whilst traditional methods have employed physical models it is apparent that more and more of the detailed design and stress analysis will be done by digital computers. The availability of high resolution interactive graphics and high quality plotters further reduce the design times involved. Several significant structures have now been constructed in Australia using the computer shapes and cutting patterns and it can be stated that these procedures have been adequately proven in practice.



LINEAR QUAD

PARABOLIC QUAD

FULL QUAD

CUBIC QUAD

FIG. 1 QUADRILATERAL SURFACE PATCHES
SHOWING REQUIRED LOCAL NODE NUMBERING

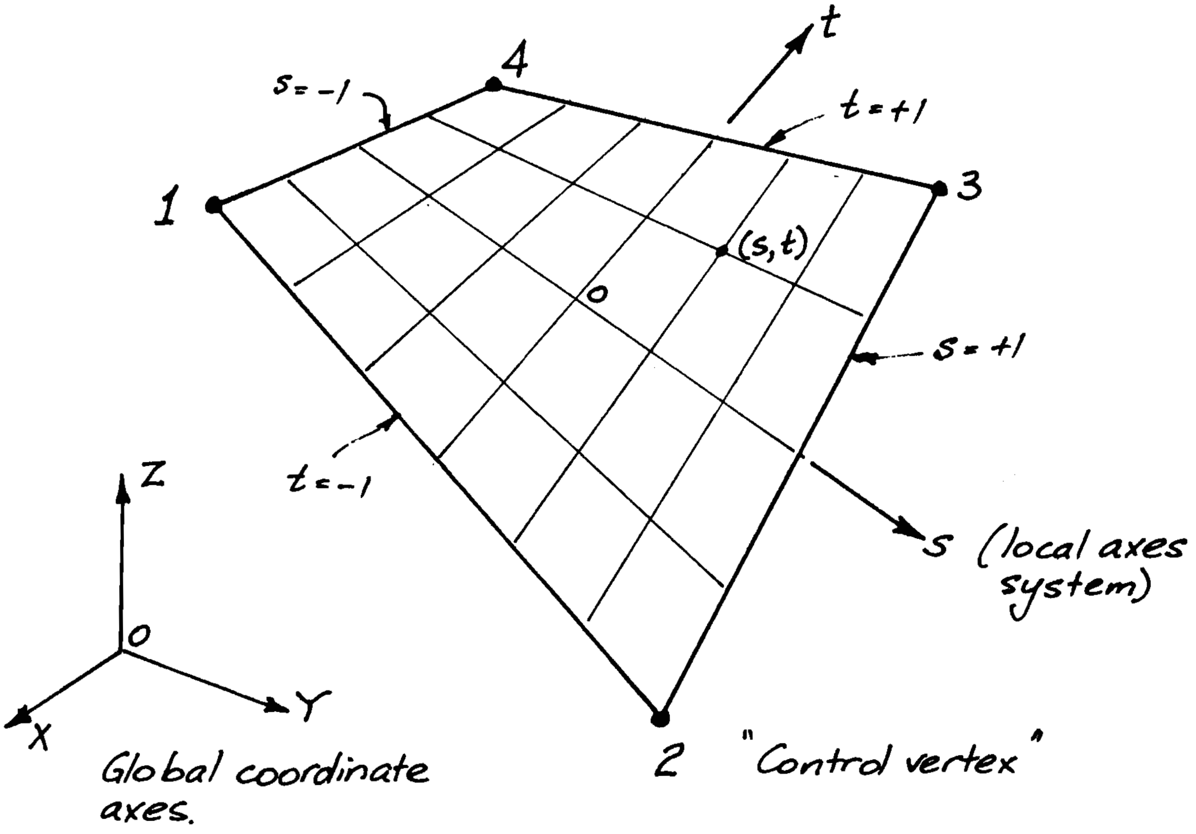


FIG 2 GENERATION OF A MESH USING THE LINEAR QUAD

$$x(s,t) = \sum_{i=1}^{4} B_i X_i$$

$$y(s,t) = \sum_{i=1}^{4} B_i Y_i$$

$$z(s,t) = \sum_{i=1}^{4} B_i Z_i$$

where B_i are the blending functions at each of the 4 control vertices.

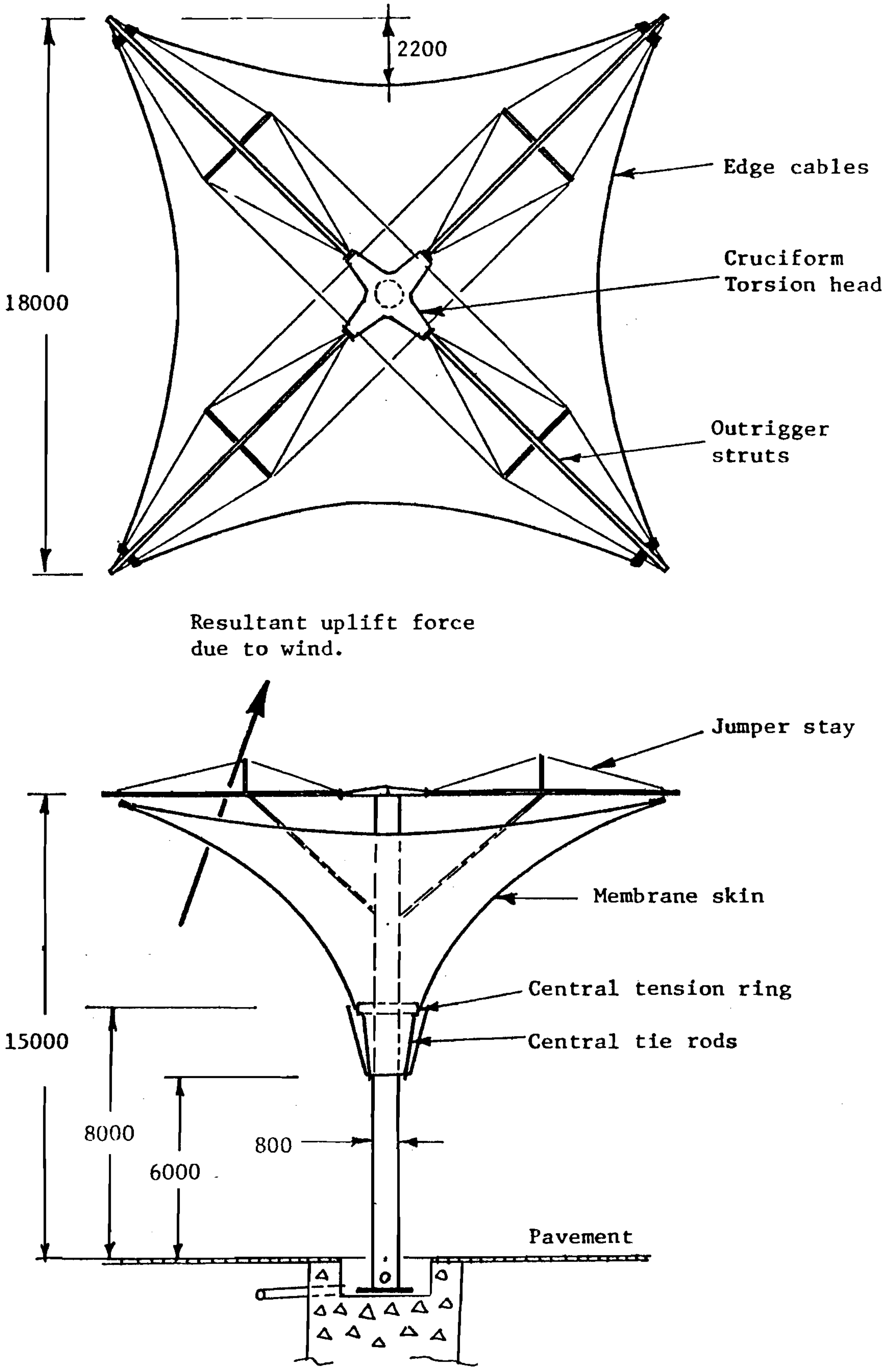


Figure 3 PLAN AND ELEVATION - QUEEN ST. SHADE STRUCTURES

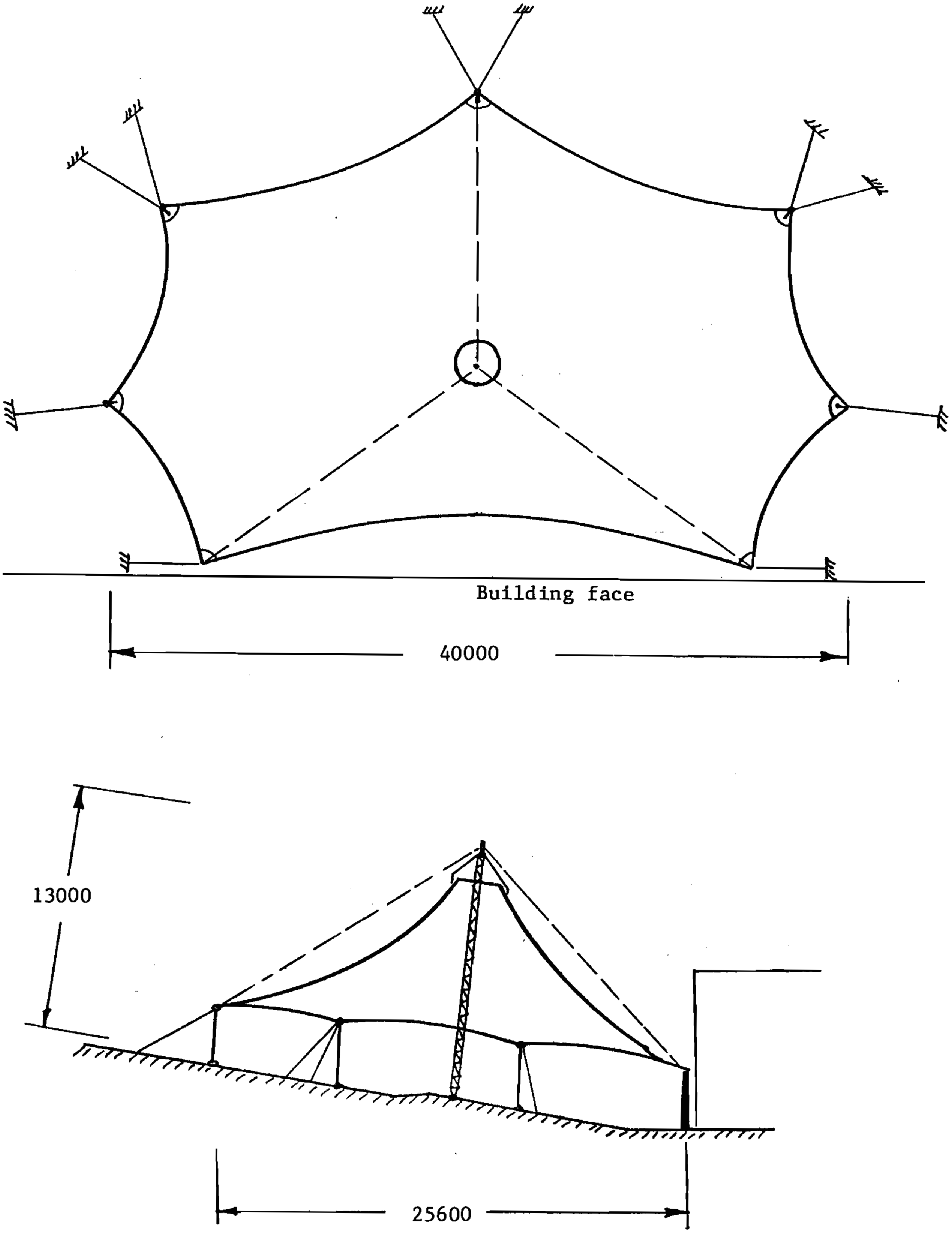


Figure 4 PLAN AND SECTION OF AMPHITHEATRE ROOF (EUROA)

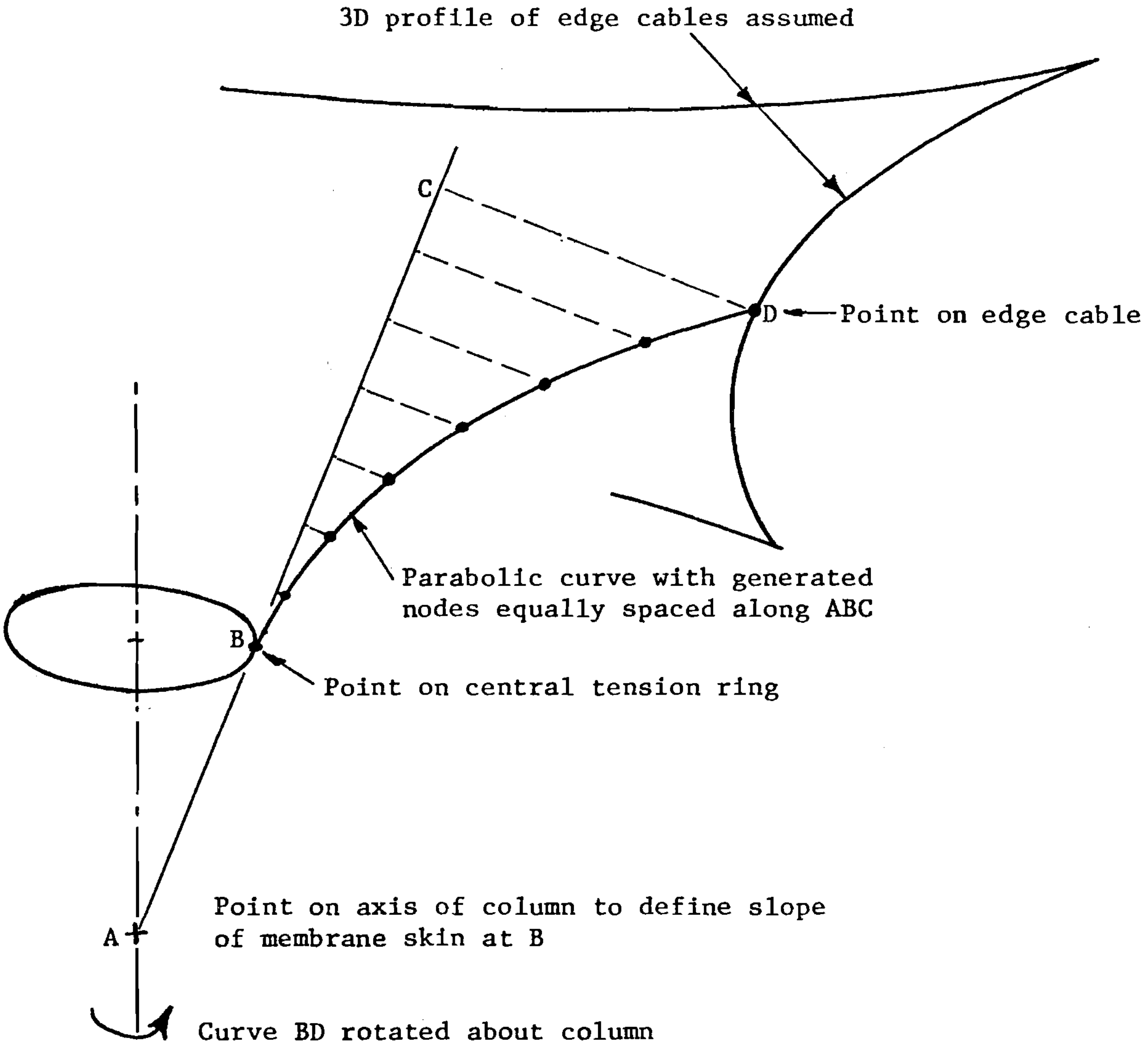


Figure 5 PROCEDURE USED TO DEFINE SURFACE

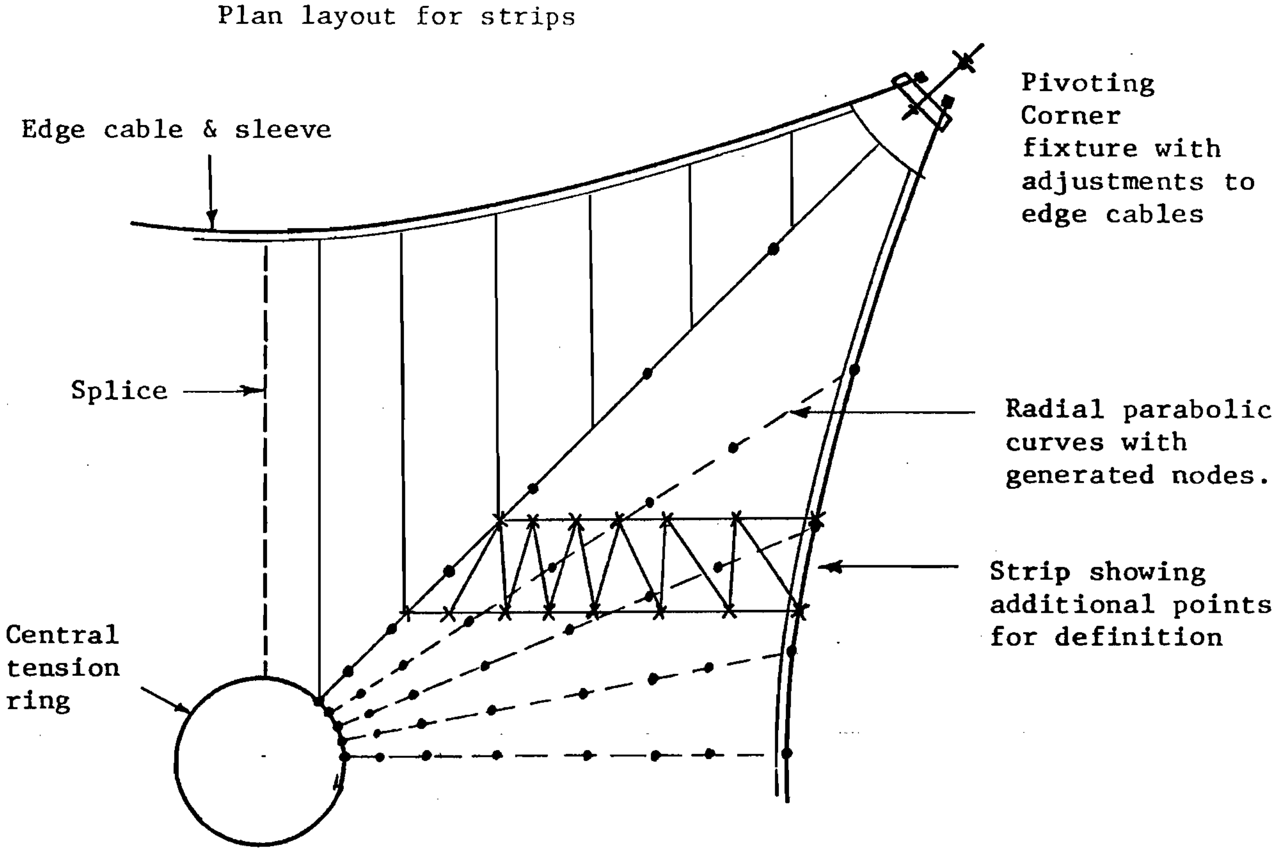


Figure 6(a) PLAN VIEW OF ADOPTED CUTTING PATTERNS

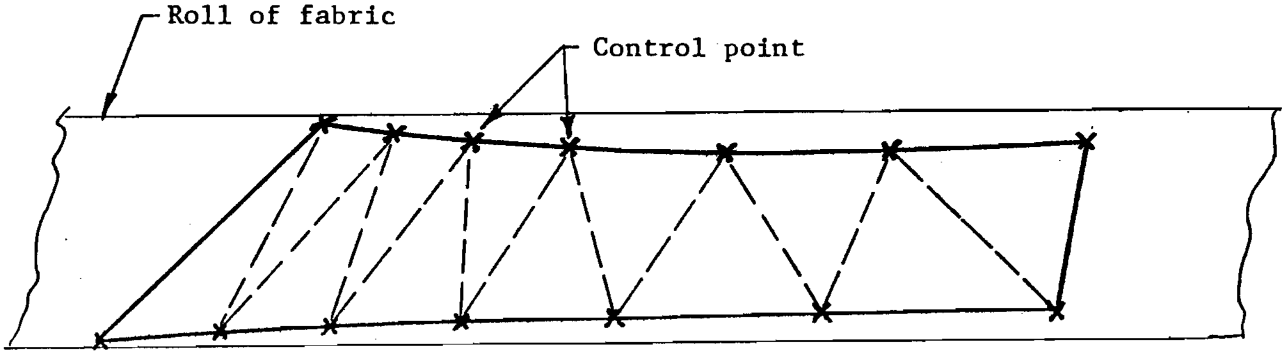
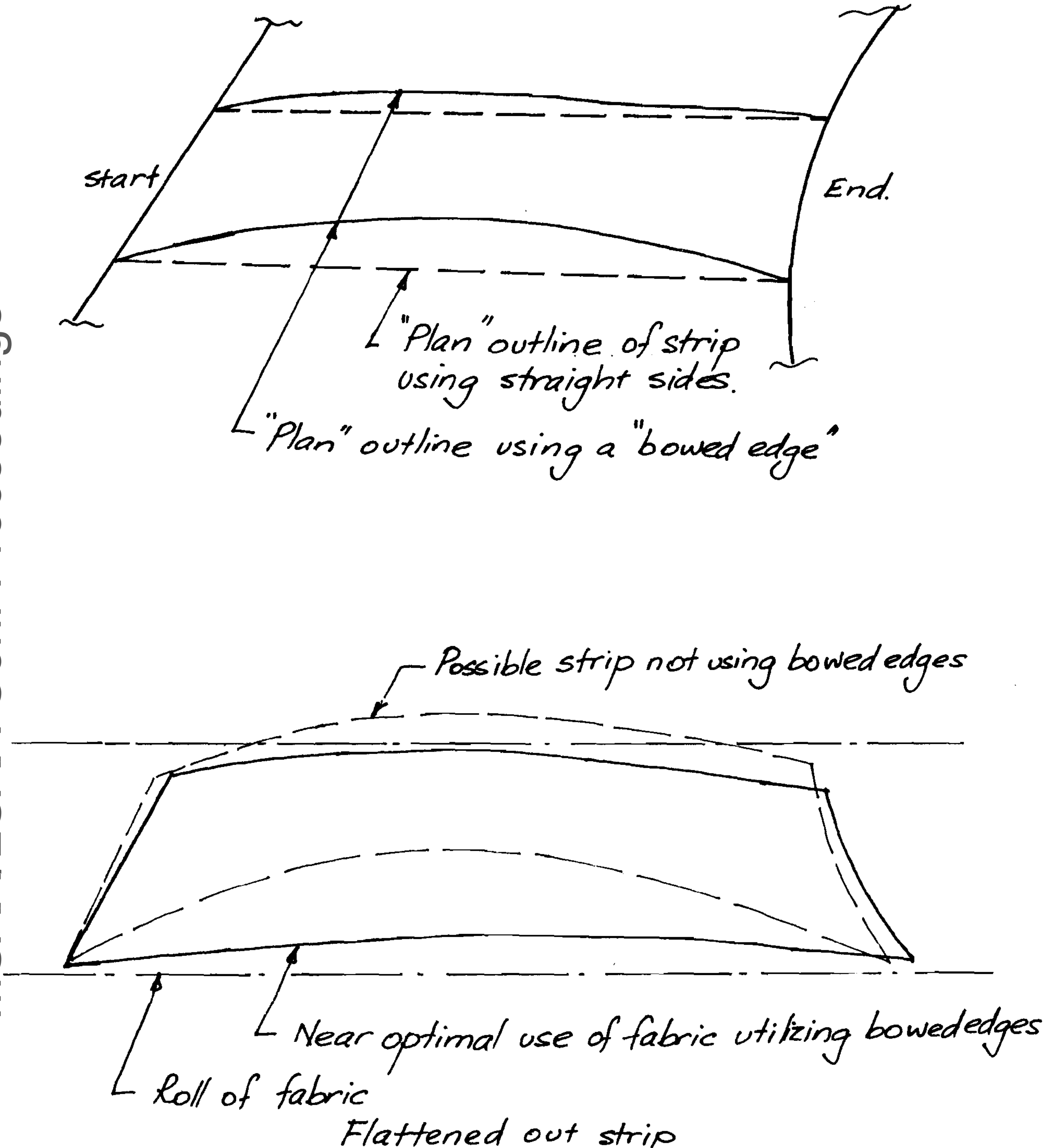


Figure 6(b) FLATTENED STRIP ON ROLL OF FABRIC



Flattened out strip

FIG 7. SOME REFINEMENTS TO CUTTING PATTERNS