LONG SPAN CABLE SUPPORTED ROOF STRUCTURES

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1. INTRODUCTION

This paper deals with long span roof structures in which a primary source of support is from a cable or from a system of cables. The paper examines various types of roofs which utilize cables and discusses the structural behaviour and the contributions made by the cable component.

2. THE BEHAVIOUR OF A SUSPENDED CABLE

In most engineering applications involving a suspended cable, the amount of sag permitted or desired is governed by the costs and appearances associated with providing the supports at each end of the cable. For small sags, the cable tension is higher and stronger anchors are required to resist the horizontal component of this force. An advantage of a small sag is that any supporting mast or side wall is shorter and hence more efficient in compression, cheaper as well as being easier to transport and erect. See Figure 1.

The great disadvantage when a suspended cable is used as part of roof system is the relatively large and unacceptable movements that occur under **non–uniform** loading. For a large span roof system, the cladding is likely to be a metal deck together with perhaps thermal and sound insulation material. The self weight of the roof, in these cases, will be generally significantly smaller than the design wind loads with large regions of a **roof** possibly being subjected to uplift forces. In these situations, the pure strength advantages of a suspended cable **cannot** be used to its full potential. See Figure 2.

2.1 Improving the Behaviour of Suspended Cable Roofs

Various techniques are used to overcome the unserviceable behaviour but most can be described as building into the roof some means of providing additional downward loads **onto** the chief load carrying suspension cable.

In conceptual terms, the simplest technique is to add more weight to the roof. Thus, the applied wind uplift forces will then be insufficient to overcome the self weight and will be a smaller component of the total load. Force variations in the cables will also be reduced **and** deflections will be reduced. See Figure 3.

A more **frequently** used strategy, however, is to build in downward loads by means of an opposing cable system which preloads the main suspension cables. Any uplift loads are able to transmit the forces to this second cable system. If the roof system is rectangular in **plan**, then a **series** of planar **semi-independent** cables can be used. Here prestressed cable "beams" or "trusses" can be used in a variety of forms as shown in Figures 4 and 7.

When the boundary of the roof can vary in height, it may be possible to adopt a true **cablenet** system in which there is an anticlastic doubly curved surface shape with two sets of orthogonal cables. When stressed into position, each set of cables will preload the other as indicated in Figure 5.

3. THE USE OF CABLES IN MEMBRANE STRUCTURES

In this conference, where the main emphasis has traditionally been the fabric or membrane component of a roof, little attention has been given to the cable **component(s)** present in the majority of the tension structures.

The fabric material needs to be prestressed to avoid flapping. This is achieved by deliberately manufacturing the panel to be smaller than the final shape. This smaller panel must be stretched, often by adjusting the lengths of cables contained in the structure. In terms of the above discussion, the fabric itself performs the role of preloading any main supporting cables as well as being the roof or environmental barrier.

Edge cables are commonly used to gather the tensile forces from the membrane and to redirect these distributed surface forces to conveniently located and isolated anchorage points at mast tops or foundation level.

Ridge and valley cables are often used to control or reduce fabric stresses as well as to influence the amount of clearance underneath a membrane structure. They permit sudden changes in fabric direction to occur, often adding to the architectural statement of the design. Such direction changes require the cable to impart a reactive force to the fabric of considerable magnitude to maintain equilibrium. This, in turn, translates into relatively large tensile forces in the cable. Fortunately a wide range of cable strengths are available as distinct from the limited range of fabric strengths.

For many practitioners, the inclusion of a ridge or valley cable in a membrane structure is often regarded as a "force-gathering" mechanism as well as a "membrane-stress reduction" mechanism. The membrane stress reductions are achieved by several mechanisms. One mechanism is that the free span of a panel of fabric is reduced because of the prescence of an intermediate ridge cable. A second mechanism is that there are often no localized stresses in the fabric owing to a concentrated fabric attachment point such as a masthead. The ridge cable spreads the attachment out over the full span of the ridge cable.

Another advantageous aspect when ridge cables are used is that the complexities of the surface shape are often reduced, leading to more efficient fabric cutting patterns, as well as fewer requirements for reinforced patches. Overall, many structures with ridge cables have a cleaner, less cluttered appearance.

A safety aspect of a tension structure incorporating fabric supporting cables between all masts is that, in the event of a major fabric tear propagating, it is less likely that the main masts will collapse. Because of this, extra external safety guys to mast tops may be omitted. Without the extra safety guys direct to mast tops, the overall shape of the membrane structure can change to adjust to a localized failure. It is likely that damage would be limited to the initial tear.

3.1 Cable Forces in Edge and Ridge Cables

The forces are determined in the first instance by simple hand calculation based on the span, the amount of sag and an estimate of the applied load. In a membrane structure, a major component to the applied load is the amount of fabric prestress imposed or desired **for** edge cables, and the sum of the components of the fabric prestress which will act in the plane taken up by a ridge cable.

Most engineers would adopt the simple formulae for a parabolic cable subjected to a uniformly distributed load expressed as a force per unit length of the clear span or

perpendicular to the chord joining the two end points of the cable. Furthermore, in most membrane structures, a good starting point would be to use the "horizontal component" of the cable tension (H). Most books dealing with cables implicitly assume that self weight of cables, or the load carried will always be acting downwards such as in a freely suspended cable (eg transmission line) or as in a suspension bridge. Such cases are usually approximated by a parabola.

Obviously in a tensioned membrane structure, the self weight of the cables and membrane is small compared to the in-plane fabric stresses and the cable will not hang in, or be stressed to form a vertical plane. In fact, due to the double curvature of the surface, the cable will not generally lie in any plane at all. Nevertheless, the maximum distance from the chord line to the cable profile can be taken as the "sag" and the "horizontal component" can be thought of as the cable force at this point of maximum sag provided that the two ends are at similar heights. See Figure **6**.

Thus an estimate of this force is given as:

$H = w(span)^2 / (8^* sag)$

Under uneven wind loads, the membrane structure will change shape in the first instance so as to achieve equilibrium of all forces, and secondly (actually at the same time) the components will stretch or shrink causing stresses to vary.

It is an extremely complex proposition to estimate this change of shape by hand and nowadays non–linear, large displacement finite element analysis programs are used to model the behaviour. All sorts of unknowns come into play such as:

- a unknown pressure distributions due to the wind and inherent gustiness.
- a unknown state of stress in the fabric considering as yet ignored long term creep behaviour.
- a unknown states of initial stresses in the fabric due to the cutting patterns and fabrication influences such as shrinkage due to welding.
- a the unknown initial lengths of adjustable cables at the time of the analysis.
- a approximations in the finite element model and the closeness of the mesh used to model the structure.

3.2 Cable Load Factors and Prestretching

Given all these unknowns, as a first step it seems that, in many cases, the use of the simple formula with an appropriate load or safety factor will suffice. For cables subjected to frequently alternating loads (eg cranes and elevators), factors of 5-6 are common. For membrane structures, where the designer certainly wishes to avoid all loss of tension in **any** portion of the fabric, the edge and ridge cables will not suffer such large force variations and any variations will be less suddenly applied as the surface will take some time to change shape. In these cases, a factor of (say) 2.5-3 would seem appropriate. This factor should be increased significantly if only the no-load, prestressed load case is considered for cable force estimates. Care is required to ensure that all fittings are selected or designed to be **as** strong or stronger that the cable itself. Due to the complex geometry and stress distribution in objects such as shackles, turnbuckles, rigging screws, swaged threaded ends etc, the factors quoted are based on ultimate loads determined by destructive tests.

It is not uncommon to subject main structural cables to "**prestretching**" to reasonably high levels of stress. This process effectively eliminates "constructional" stretch in the cable.

Bedding down of the wire strands around a fibre core, for example, will cause the cable to stretch much more on the first loading application than on subsequent loadings. In engineering terms, the elastic modulus can be increased by perhaps 30–40 percent when prestretching is used. Another aspect of prestretching is that cable lengths are often marked out on the loaded cable. Thus, this indicates that the prestretching load will be comparable in magnitude to the forces generated by the fabric under prestress conditions.

4. PRETENSIONED CABLE BEAM AND TRUSS SYSTEMS

Various forms of these planar systems are used (see also Figure 7:)

- 1. a convex cable beam system with struts,
- 2. a concave cable beam system with ties or struts,
- 3. a convex-concave system with struts,
- 4. a parallel chord cable truss system with web diagonals and struts,
- 5. a concave beam with diagonal ties.

For a roof, the loadings on adjacent trusses will vary and it may be necessary to examine the differential movements between trusses. To protect the cladding and fasteners, stiffer **purlins**, or even stiffening trusses may need to be incorporated between adjacent cable trusses.

It is also possible to adapt these forms to a radial system.

4.1 Prestressing Techniques

Several main methods are possible with the basic forms:

- 1. to shorten the lengths of the backstays by means of hydraulic jacks or threaded rods after the main span cable truss system is in place,
- 2. to adjust the lengths of the intermediate struts or ties,
- 3. to **construct** the main span to a smaller length and to anchor this separate **truss** to one end mast and then jack against the other mast to attach the far end to the second mast without touching the backstays,
- 4. to adjust each separate main cable at the mastheads,
- 5. to jack up the masts.

Which particular method is used will depend on practical considerations such as access to jacking points, whether two or more points need adjustment simultaneously and the connection design in order to accommodate the jacking equipment.

The choice of whether jacking or screw threads are used will be governed by the method used to specify the level of initial prestress. Thus, if a required initial force is desired then jacking with a calibrated pressure gauge seems logical. Alternatively, prescribing actual initial cable lengths permits the prestress forces to be derived from a knowledge of the amounts of thread taken up. The latter point, however, would be sensitive to the accuracy of setting out the masts and anchorage locations and would seem to have less control compared to using the jacking forces.

Another aspect needs to be borne in mind if more than one truss or jacking point is involved. By adjusting one cable length this may cause other cable forces to fluctuate and the complete jacking operation may require taking up a little at a time, working systematically around the structure. Given this, careful planning is required to reduce the number of jacking points to a minimum.

4.2 Prestressed Cable Gridworks

For some roof plan shapes, it is possible to have two sets of intersecting orthogonal planar cable truss systems of the forms described above. For nonradial systems there would be a considerable variation in geometry when the concave, convex or **concave–convex** trusses were used. The resulting doubly curved roof surface may prove more difficult to clad.

These forms are suited to the parallel chord (or nearly so) trusses and a rectangular plan with columns and backstays permitted on all sides.

5. PRESTRESSED CABLE NET STRUCTURES

The use of two orthogonal connected cable systems results in a doubly curved net structure with each cable set preloading the other direction. In fact, with both sets of cables having a similar importance, the behaviour is not dissimilar to a membrane structure. The cables would be spaced at fairly regular intervals (say 0.5m to 3m) to suit the cladding material which would rarely be prestressed in itself. Because the cables can be made of almost any diameter the effective prestress in the surface expressed as a force per unit length can be substantially higher than for a membrane structure. As a result, the curvatures of the surface are smaller giving larger spans for the same rise in the shapes.

As with membrane structures, the **cablenet** can be terminated at stronger edge cables, edge beams, perimeter ring beams etc. and can also incorporate stronger ridge and valley cables.

The deflections of a **cablenet** under wind loading will be governed by the same criteria as a membrane structure, namely:

- 1. the amount of curvature in the roof surface,
- 2. the level of prestress in the cables,
- 3. the stiffness of the cables (ie the cross sectional area and type of cables used wire rope with a fibre core, wire rope core, parallel strand etc),
- 4. the rigidity of the supporting structure and edges.
- 5. whether any cable becomes slack during the deformation.

As with membrane structures, many analysis problems are similar but perhaps more readily modelled, for example:

- the effects of prestress are vital to the behaviour, and any analysis must be able to account for this – for example by specification of different "initial lengths" to the node to node calculated lengths.
- 2. the effects of possible length adjustments by either mechanical change of length (eg threaded rods without regard to forces) or by jacking (eg unknown length variation to reach a prescribed cable force).
- 3. the effects of large out of surface displacements so that the **cablenet** may assume **a** deformed geometry in equilibrium with the applied external loads such as wind.
- 4. the ability to account for some cable elements becoming slack under loading.
- 5. the ability to account for long cables such as mast guys sagging and maintaining a tensile force rather than simply being ignored if the chord length becomes **smaller** than the initial length.
- 6. the ability to be able to perform a form-finding exercise.
- 7. the ability, if required, to superimpose the stiffness of the cladding.
- 8. the ability to generate loadings normal to the surface as well as purely gravity or projected loads in the orthogonal directions.

Other similarities exist with membrane structures, and in fact many membrane structures were originally modelled both physically and analytically as a cable net system because adequate material models for the membrane material were not available, and are still not often used. The two types are extremely lightweight (ignoring the substantial guy anchorages) but have enormous inbuilt damping against vibration. Both types would move relatively large distances normal to the surface and in doing so have to push against large masses of air, the major (but unknown) contributor to the damping. It is felt that, provided the surface cannot develop localized slack areas, this damping will always be present and will help protect the structure.

In some **cablenet** structures, additional cables have been used which are attached to a connection within the main surface and are anchored to a point away from the surface to provide substantial out-of-plane resistance to movement (at least in one direction). An example of this is in the first significant **cablenet** structure, the North Carolina State Fair Arena at Raleigh, USA built in **1953.** Incidentally, perhaps the next most famous early structure was the Sidney Myer Music Bowl in Melbourne. Much of the analysis for this project was done with the aid of physical models.

6. CABLE STAYED ROOF SYSTEMS

These structures can be thought of as a three dimensional form of a cable–stayed bridge. Superficially, the bridgelroof is supported vertically at a limited number of points by straight cables or rods coming from higher masts. The bridgelroof will have a strong structural element spanning between these pickup points which is capable of resisting both bending and induced compression loads as a result of the horizontal component of the cable tension.

For medium spans of bridges (say 200-400 metres) they are much more economical than a true suspension bridge. Cables are stock sizes of relatively short lengths, and may permit the use of standard prestressing jacks. The bridge can be constructed without extensive falsework or long delays whilst a main suspension cable is spun in place (up to 2 years **for** a large bridge). The deck itself can be cantilevered beyond the last cable pickup point to the next attachment which can be jacked independently.

One major advantage of a cable stayed bridge or roof is that some designs enable the difficult, very large horizontal component of the cable tensions to be taken by an opposing compressive force in the deck itself. In turn, this may eliminate the need for major side anchorages which are then simply a vertical support (tension or compression). Thus, quite poor soil conditions do not pose a major problem for a cable stayed system.

For roof structures, a large area can be covered by designing, in effect, numerous cable stayed bridges, side by side. The "rigid" continuous deck of the bridge need not be present in a roof where the panels between cable pickups can be lifted into position by cranes working from a clear area on the floor of the building. For speed of erection, the connections **can** be pinned with perhaps the possibility of providing continuity at a later stage after cable adjustments are made. There will be a need most likely to have members with continuity between adjacent "bridges" in the roof. If the spacing between adjacent "bridges" is increased, then these intermediate members may also be supported by inclined cables to the masts.

As distinct from a road or rail bridge where the main deck would only have a gradual curved profile, a roof may be segmented with sudden changes in slope between cable pick up points.

For some roof structures, the resulting light weight might not be enough to outweigh wind uplift forces. In these cases, some designs incorporate an opposing cable system with compression struts between the two so as to resist uplift without severe distortions to the roof.

In Australia, significant cable supported roofs have been constructed such as the Homebush Bay Sports Centre and the Exhibition Building at Darling Harbour, both in Sydney and the Sports Centre in Canberra.

6.1 Refinements to Cable Stayed Roofs

To reduce the amount of strength in the deck or bending elements of a roof, it is possible to add additional cables in a form which transmits the local loads directly into the masts rather than by partially spreading any localized loads out before they reach a cable (see Figures 7d and 7e). It can be seen that a concentrated load at a pickup point is quickly dissipated to the masts. This arrangement can be applied to roof systems as well with the main members almost acting as a resisting arch.

Several preliminary studies indicate that significant spans for roof structures (say 80–100+ metres) will be possible using standard UB or even large RHS sections. For footbridges, the benefits do not appear to be that obvious where a substantial live load may be placed only between pickup points instead of at a pickup point. The smaller slope and longer length of the "auxiliary" support cable makes it less efficient. The larger number of cables and the fact that more cables meet at a point will make the detailing more difficult especially if means for cable adjustments are required. In many applications, it would seem cheaper to adopt the standard forms and use slightly larger deck members.

7. THEORETICAL BASIS FOR ANALYSIS

When large, or significant displacements are involved, the normal equations of force equilibrium at each nodal degree of freedom should be formulated in the deformed configuration of the structure. The normal linear stiffness equation P = KD between applied loads and displacements must take into account changes in geometry and the effects of strain-displacements on the stiffness matrices of the elements. In essence, the member stiffness matrix, **k**, is replaced by: $k = k_E + k_G$ where the first term is the normal elastic stiffness matrix which is a function of the elastic properties of the material and the geometric cross section properties. The second term is a function of the **geometry** of the member (its length), and the internal forces present in the member, and in particular the axial force. It is termed the geometric stiffness matrix.

Since the geometric stiffness matrix depends on the internal forces present, it will **require** that either a good estimate is available – which defeats the purpose of the analysis – or, that we go about solving the equations in an iterative manner. For the first step, we could **assume** the internal forces to be as prescribed, or as zero and perform an analysis. For zero internal initial forces, the geometric stiffness matrix is zero and the first step would correspond to **a normal** linear stiffness analysis. This first step will provide values for node displacements as well as the member forces.

If we were to take all the calculated internal forces in the deformed position, we would find that equilibrium was not satisfied at the nodes – there would be some "**out** of balance" nodal forces present. However, by performing another step, and incorporating the geometric stiffness matrices for members, based on the force estimates obtained, and calculated **at** the displaced position, we would expect to reduce these out of balance forces to a smaller

amount. In fact, a normal non–linear analysis may take from 3–20 steps to converge to an acceptable state. This state is described as being when the maximum absolute value of any out of balance force component is less than a prescribed value, and/or the maximum absolute value of any incremental displacement component is less than a prescribed value.

7.1 Geometric Stiffness Matrices for Cables and Beams

For two dimensional cases, the combined member force-displacement equation for a pin-ended member such as a cable segment or truss element is given as:

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \frac{EA_1}{L} \begin{bmatrix} 10 - 10 \\ 00 & 00 \\ -10 & 10 \\ 00 & 00 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \frac{F}{L} \begin{bmatrix} 0 & 00 & 0 \\ 0 & 10 - 1 \\ 0 & 00 & 0 \\ 0 - 10 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

For flexural elements in two dimensions, the equations are:

S_1		$\int AL^2/I$	0	0	$-AL^2/l$	0	0	[<i>u</i> ₁]		0 0	0	0	0	0 7	[<i>u</i> ₁]
S_2	EI	0	12	6L	0	- 12	6L	<i>u</i> 2	и ₂ и3 <i>F</i>	0 6/5	<i>L</i> /10	0	-6/5	<i>L</i> /10	<i>u</i> 2
S_3		0	6L	$4L^2$	0	- 6L	$2L^2$	из		0 <i>L/</i> 10	$2L^2/15$	50-	- <i>L</i> /10	$-L^{2}/30$	из
S 4	$=\overline{L^3}$	$-AL^2/$	0	0	AL^2/I	0	0	и4	$+\overline{L}$	0 0	0	0	. 0	0	u4
S5		0	- 12	- 6L	0	12	$2 - 6L u_5$		0-6/5	- <i>L</i> /10	0 (6/5	- <i>L</i> /10	u5	
S_6		0	6 L	$2L^2$	0	- 6L	$4L^2$	<u>u</u> 6		0 <i>L</i> /10	$-L^{2}/30$	00-	- <i>L</i> /10	$2L^2/15$	Lug

As can be seen, the effect of a compressive axial force (ie F is negative) will decrease the member stiffness. In **fact** the member will buckle when the combined stiffness reaches zero in any component. In a similar fashion, for several members connected at a node, the stability of the node may be in doubt if the combined assembled stiffness terms are zero or negative.

Because of the dependence of the combined member stiffness matrix on the internal forces, and the deformed shape, a large displacement, nonlinear analysis can only be effectively carried out with one loading condition at a time. This is a severe imposition since there **will** be several load cases to examine in detail. As each load case will require numerous iterations to converge, the computational effort is not trivial.

8. SOFTWARE AVAILABLE

Several software packages are available for analysing the non-linear behaviour of a cable supported roof system. They may be considered as a "subset" of the normal tensioned membrane structure and hence be handled by systems such as FABDES. Alternatively, structures which are composed only of beams, pin ended struts and cables may be better considered with the interactive program CABEAM. Both these systems are available commercially from the writer.

9. CONCLUDING REMARKS

This paper has introduced the topic of large span cable supported roof structures and outlined the main features of their behaviour. It has drawn attention to the similarities between cablenets in particular and the tensioned membrane structures. With the **large costs** associated with the design and fabrication of membrane structures, the use of more conventional cladding with a cable supported structure is a viable alternative, albeit not **SO** exciting in its geometric and aesthetic appeal.

9. REFERENCES

- [1] D. Jawerth, 'The Jawerth Prestressed Suspension Roof of the Sports Hall in Varnamo, Sweden", **WIRE** No. 51. February 1961.
- [2] F. Otto "Tensile Structures", MIT Press, Cambridge 1962
- [3] N. Esquillan and Y. Saillard, "Hanging Roofs", North Holland 1963
- [4] A. Pugsley, "The Theory of Suspension Bridges", Edward Arnold, London1968.
- [5] Subcommittee on Cable–Suspended Structures, "Cable–Suspended Roof Construction State-of-the–Art", Journal of Structural Division, ASCE, vol 97, no ST6, June 1971.
- [6] L Glaeser, "The Work of Frei Otto", New York Graphic Society, New York, 1972
- [7] P. Krishna, "Cable Suspended Roofs", McGraw–Hill, 1978
- [8] M. H. Irvine, "Cable Structures", MIT Press, Cambridge, Mass., 1981.
- [9] J. Thornton, 'The Design and Construction of Cable–Stayed Roofs", TheStructural Engineer, Vol. 62A, No. 9, September 1984.
- [10] K. A. Bond and R. Hough, "Analysis and Design of a Long Span Singly-Curved Cable Roof" Civil Engineering Trans., I.E. Aust., Vol.CE25, No.1, February 1983.
- [11] H. A. Buchholdt, "Introduction to Cable Roof Structures", Cambridge University Press, Cambridge, 1985
- [12] J. Quinn, "National Athletics Stadium, ACT (Australia)", IABSE Structures Periodica Vol 2,1980
- [13] 'The State Sports Centre", Constructional Review Vol. 58, No. 2, May 1985.
- [14] B. O'hea, 'The Roof of the darling harbour Exhibition Centre", AISC Steelin Structure Seminar, August 1987.
- [15] M. S. Troitsky, "Cable–Stayed Bridges", Crosby Lockwood Staples, 1988.
- [16] J. W. Leonard, "Tension Structures", McGraw-Hill, 1988.
- [17] Kneen, P.W. and Thorvaldson, F., "Burrendong Cable Net Shade Structure", Landscape Australia, 1980.
- [18] Kneen, P.W. "Fabdes A Program for the Design of Fabric Structures", University of New South Wales (1987).
- [19] Kneen, P.W. "cabeam A Nonlinear Program for the Analysis of Beam and Cable Structures", School of Civil Engineering, UNSW, 1988.
- [20] Kneen, P.W. Examples of the Non Linear Analysis of Beam and Cable Structures using the CABEAM Program. School of Civil Engineering, UNSW, 1988.
- [21] Kneen, P.W. "Postlisa A Program for the Post Processing of Analysis Results of Lisa and Cabeam Programs", UNSW 1988.
- [22] D. Carolan, "Cable Structures", M.Eng.Sc. Thesis, School of Civil Engineering, UNSW, 1989.



Figure 1 Simple Cable Under Gravity Loads Imposes Large Horizontal Reactions.



Figure 3 Simple cable with smaller sag and unevenly distributed loads produces smaller deflections







Figure 7b Side View of a "Tenca" Cable Truss



Figure 7d Load Path Through a Simple Funicular Cable Truss



Figure 7e Side Elevation of Membrane Roof with Funicular Cable Truss

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