THEORY OF CURVATURE AND LOADING FOR MEMBRANE STRUCTURES

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INTRODUCTION

Membrane Structures are distinctive for their shape and form. The flowing lines of their surfaces and variations in elevation around the perimeter of their shapes present to the observer features which stand out from most other structural forms. Do these wonderful shapes have some significance in the performance of these structures and so although they appear to be free form in expression, are they constrained by exact formulae and set solutions? Or do we as an industry deliberately make our structures different so that they are easier to market? Are we working harder than other areas of the construction industry, because we are blending nature with science, combining versatility and rigidity and closing the differences between architect and engineer?

This paper is to investigate the main issues in defining the shape and curvature of a membrane structure. To look at what is governing the stresses and economics when we vary the shape of a structure and then to summarise the loads that are applied.

MATERIAL PROPERTIES

By definition a membrane structure uses a thin membrane as its major structural element. So this material has particular properties that have to be accounted for in a structure.

The most obvious of these is that it can sustain tensile loads and some planar shear loads with no capacity to resist compression, bending or transverse shear.

Secondly, it is a weak material with design strength (working loads) in the order of 5 to 20 MPa which compares to steel with a design strength in the order of 100-200 MPa. So steel is in the order of 10-20 times as strong as the fabric.

The other difficulties with the fabric are its non-linearity of stress/strain properties, different properties in different directions and long term loading patterns.

As the fabric is loaded and passes through a series of load cycles the fabric becomes stiffer and behaves in a more linear manner. The fabric will normally have different properties in the warp and the weft directions. This will depend on the type of the weave for the fabric and also its strength characteristics in each direction. Long term loading of the fabric will effect the loss of strength of the fabric with time. The higher the long term load or prestress the less strength the membrane will have in time.

For these 5 reasons, a major criterion in the design of tension members is to reduce the loads in the membrane. By doing this the tension stresses in the fabric and the other material properties which effect its performance can be kept within acceptable limits.

STRUCTURAL PRINCIPLES

The strength of a structural element is normally defined by its cross section properties, ie. moment of inertia and section modulus. When we look at a thin membrane you can see there is little opportunity for improving the poor section properties of material that is only 1mm thick.

However, the strength of a tension membrane lies in the structural depth of the complete structure. (See figure 1 & 2). As the depth increases the loads in the membrane decrease. This depth for any given span can relate back to the radius of curvature of the membrane (see figure 3).

The curvature of the surface of a membrane structure is critical in its design to maintain a viable solution, with the radius x the pressure = tension load in the membrane material. In fact the way to define a point geometrically on a curved surface is by its X, Y and Z co-ordinate and the radii of curvature in two directions. Lets look at this closely to see what actions are occurring on an element on the surface of a tension membrane. If we assume an element with sides parallel to the X & Y axis cut out of a curved member with radii of curvature Rx and Ry and twist in the X Y directions (figure 4).

A pressure p is applied to this element in the z direction. The tension in the membrane will be created in the two directions and the proportion of the load is proportional to the two radii.

The shear in the element S can be resolved into Sx (or Sy) and Sz. The Sz component helps support the loads normal to the membrane. The twist txy of the surface can be defined as the rate of change of slope in the x direction as you move along a boundary parallel to the y axis. This quantity establishes the twist of the element.

To combine all these elements into a single formula it is necessary to redefine the twist txy as a 'radius of twist'

$$Rxy = \frac{1}{txy}$$

$$p = \frac{Tx}{Rx} + \frac{Ty}{Ry} + \frac{2S}{Rxy}$$

(p is load/unit area and T and S have units of load/unit length)

From this formula the importance of the curvature upon the load curving capacity of a thin membrane becomes obvious.

Some other important facts about different types of membrane structures can be simply derived.

If the membrane is flat than Rx = Ry = and Rxy = ∞ (infinity) and so p = 0, proving that a flat membrane cannot resist loads normal to the surface.

If the element is so orientated that the x and y axis are the principal directions 1, 2 there is no twist in the element xy and so there is no shear, so:

$$P = \frac{T1}{R1} + \frac{T2}{R2}$$

For a cylindrical shape the element is flat in one of the principal directions say R1 = ∞ and the normal pressure becomes:

$$P = \frac{T2}{R2}$$

For a spherical surface R1 = R2 = R

$$P = \frac{2T}{R} \quad \text{or} \quad T = \frac{PR}{2}$$

So this shows the reliance of the membrane stresses on the curvature of the surface.

PRETENSION

The other limitation mentioned earlier about materials properties was that the membrane can only carry tension loads and does not work in compression. A fabric going into compression will carry no load, will buckle and wrinkle and in cyclic loads cause higher stresses than normal in the fabric as it jumps in and out of tension.

To stabilise the structure the membrane is pretensioned to a load that will be greater than any compression loads likely to occur in the life of the structure.

This pretensioning occurs in two ways:

1. By applying the tension loads to the open boundary of the membrane, ie. edge cables or turn buckles on membrane plates or back stays to columns this is the normal case in a 'tent structure'.

2. By applying a uniform pressure to the surface of a closed membrane to tension the fabric, this is called a 'pneumatic structure'.

Applying this information to original equation for the principal direction;

$$p = \frac{T1}{R1} + \frac{T2}{R2}$$

in a pretensioned but unloaded membrane, then the pressure p = 0 (figure 5)

$$\frac{T1}{R1} = \frac{-T2}{Rs} \quad \text{or} \quad \frac{T1}{T2} = \frac{-R1}{R2}$$

Since T1 and T2 only be the same sign, ie. tension only, then R1 and R2 must be of opposite sign.

So in an open membrane structure with applied external forces, the principal curvatures must be of opposite sign.

This then limits the types of shapes that can be used as an open membrane, to the shapes which can be classified as saddle or anticlastic surfaces.

The pneumatic structure has an applied pressure to pretension it, so $p \neq 0$ so T1 and T2 will be the same sign and R1 and R2 must also be of the same sign, ie. the same direction. This defines the allowable shapes for pneumatic structure which are domical or a synclastic shape (see figure 6).

EDGE CABLES

The theory applied to the membrane material also applies to the edge cables that are used to apply the external forces to the boundaries of an open membrane. By reducing the radius in the edge cable or increasing the cutout the force in the cable to apply the pretension or resist the wind loads is also reduced. (figure 7).

A graph plotting cutout versus cable load for an edge cable that has to resist a fabric loading reveals the advantages of increasing the cutout. A small increase in the cutout or reduction in radius can greatly reduce the cable load and also the reactions at the corners of the structure (figure 8).

PROBLEM AREAS

However, up to this point all the discussion has been on increasing curvature in the membrane or the edge cable to reduce tension loads, but there are disadvantages if this is taken to the extreme, and a balance has to be maintained to achieve the correct solution.

An obvious effect of increasing the curvature in a structure is to increase the amount of material. At some point it can become pointless to reduce the membrane stresses because the amount of material is also increasing, (figure 8).

The next detrimental effect caused by increasing curvature especially for edge cables is a loss of coverage by the tension membrane. Increasing the cutout means less shade or less rain protection for the plan areas under the structure which in most cases is the main function of the structure, (figure 9).

The other area where excess curvature can be a problem to the final structure is in the patterning of the fabric. The final curved shape has to be made out of flat pieces of material. This is not an exact mathematical process and a balance of a series of items has to be considered to achieve the best solution.

Ideally a large number of thin strips of material should be used to define the curved shape, but this increases the amount of wastage of the fabric from a standard roll width and also increases the amount of welding during fabrication. Trying to use a small number of wide strips to define a curved surface can lead to a fabric which cannot be pulled out into a smooth shape leaving areas with wrinkles and little prestress loads.

Fortunately the fabric industry has developed in such a way that the majority of the supplied width of fabric can be utilised in the development of membrane shapes.

FLAT PANELS

Up to now in this paper the use of flat panels has been treated as a structural disaster with little likelihood of success, but of course there are many applications where small flat panels are used, truck sidings, awnings and signs are just some of these applications. In these situations the membrane material will deflect significantly under load to produce some curvature and therefore structural depth to resist applied loads. As the overall dimensions of these flat panels are small the deflections to produce this curvature is also small.

Longer flat panels can only be sustained with large prestress in the membrane material to reduce deflections under load.

LOADING

The design loads applied to a tension membrane will have a large influence in the choice of shape and curvature (figure 10).

The self weight of the membrane is the smallest load applied and is the order of 0.02kPa the only time this load is of concern is during erection when large pieces of fabric may be lifted in single pieces held at infrequent positions giving high local loads.

The prestress load on the fabric will vary depending on the shape and type of membrane. This load is normally in the range of 1 to 2.5 kn/m. The higher the prestress load the less deflections under live and wind loads.

Live loads on a tension membrane are infrequent as it is unlikely to ever be loaded by 0.25 kPa as required by Standards. For example a 10m x 10m Hyper would require 2500 kg of load or 30 people standing on it to achieve this type of load.

The major loading on any membrane structure in Australasia and South East Asia is the wind load. The pressure on a structure is proportional to the velocity of the wind squared, so the design pressure can vary significantly. For example design wind in Jakarta and Singapore can be in the order of 25 to 30 m/sec giving a pressure of approximately 0.4 kPa. Whereas in high risk cyclonic areas such as Cairns or Hong Kong, wind speeds up to 60 m/sec are possible resulting in design pressures in the area of 2.2 kPa. So wind loads can vary by up to a factor of 5.5 depending on their location.

Earthquake loads are generally not a problem as the tension membrane is light construction made of ductile materials.

Snow loads apply to only small portions of New Zealand and Australia. However, these loads can be significant in the design of a structure as they can be in the order of 0.5 to 3 kPa. Also, the shape of a membrane structure becomes critical to avoid snow drifts which can increase these loads by a factor of 2. Steep sloping structures can be employed to stop snow build-up.

This variation of loads means that the most economic solution to design problem will vary from location to location as the applicable load varies.

CONCLUSION

The trademarks of a tension membrane are its freeform and flexibility. This paper has set out to demonstrate that these features are not only aesthetic advantages of the structure but an integral part of its strength and stability. By understanding how the curvature dictates the loads with the membrane we can use it to our advantage and produce effective solutions to design situations.

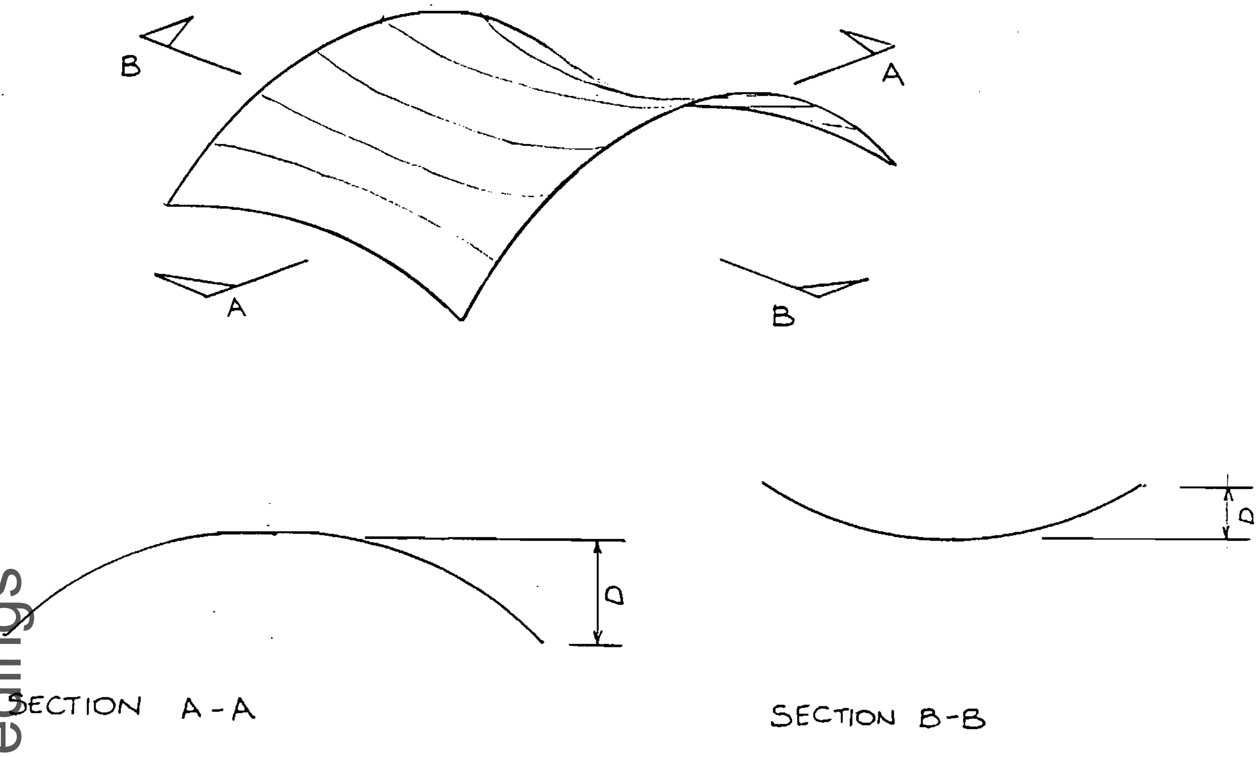


Fig (1)

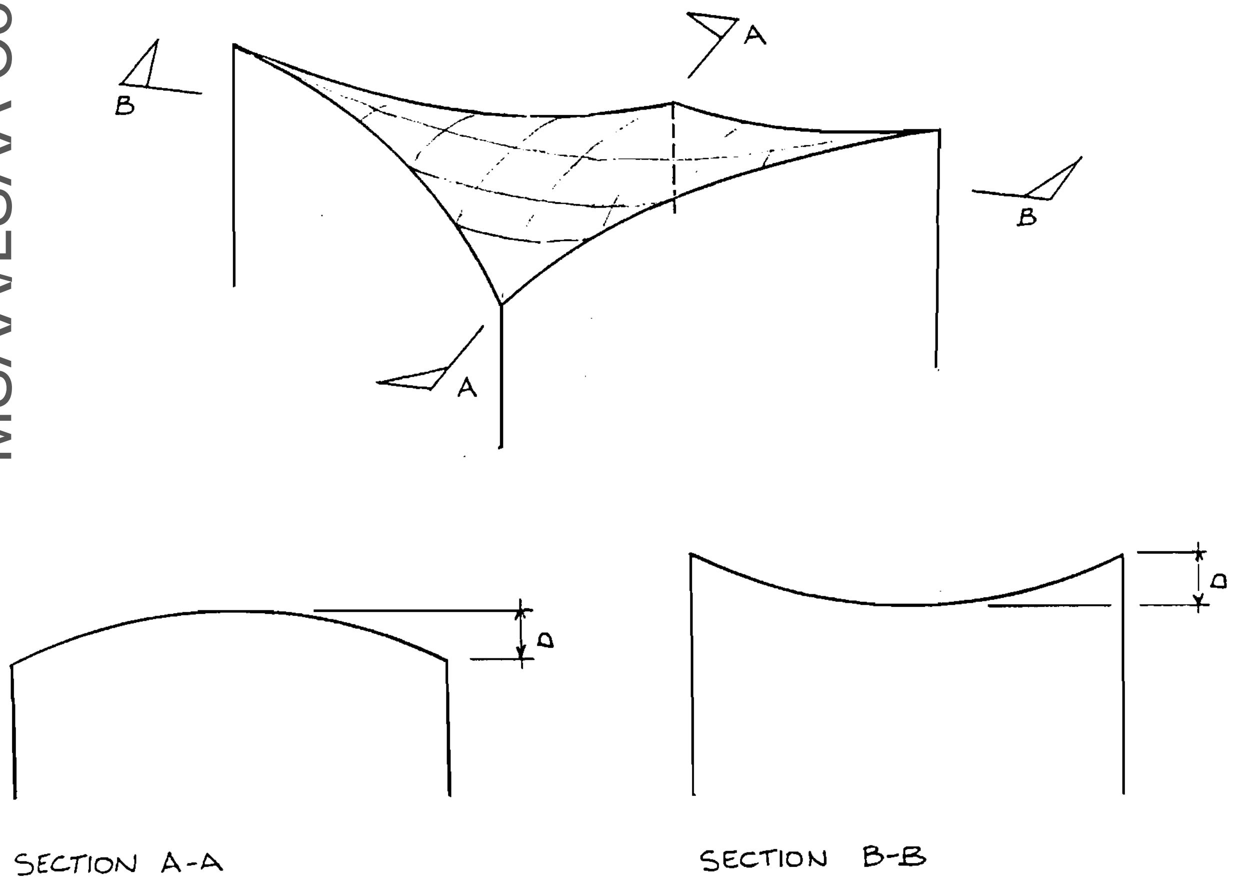
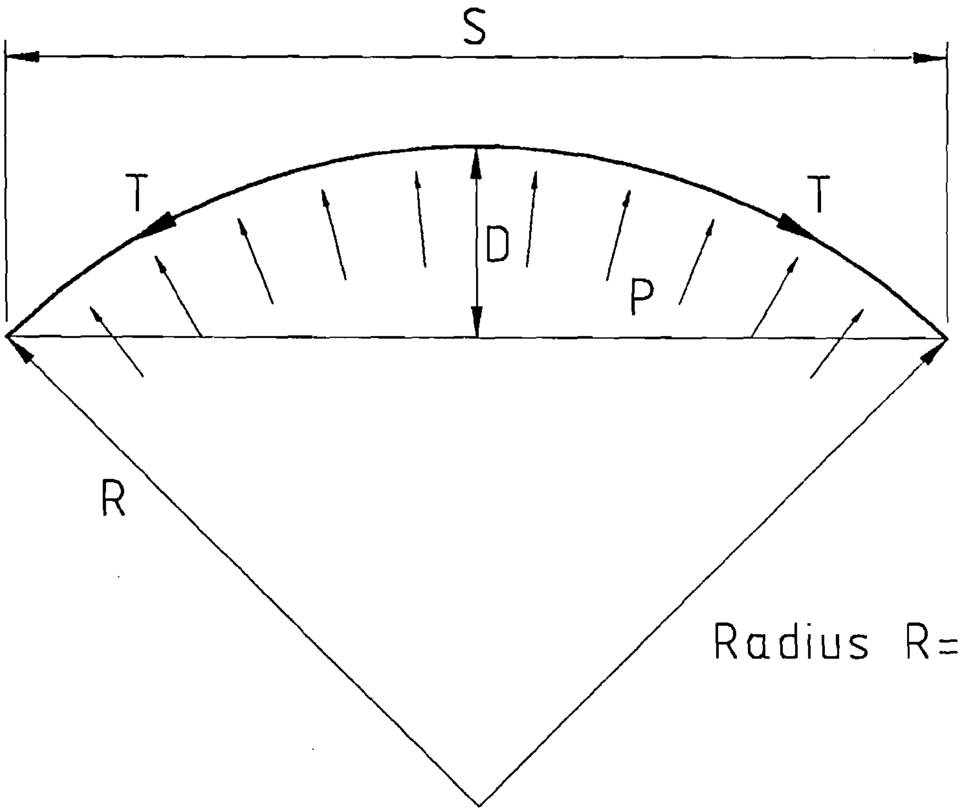


Fig (2)



Radius $R = \dfrac{\left(\dfrac{S}{2}\right)^2 + D^2}{2D}$

$T = P \times R$

Fig (3)

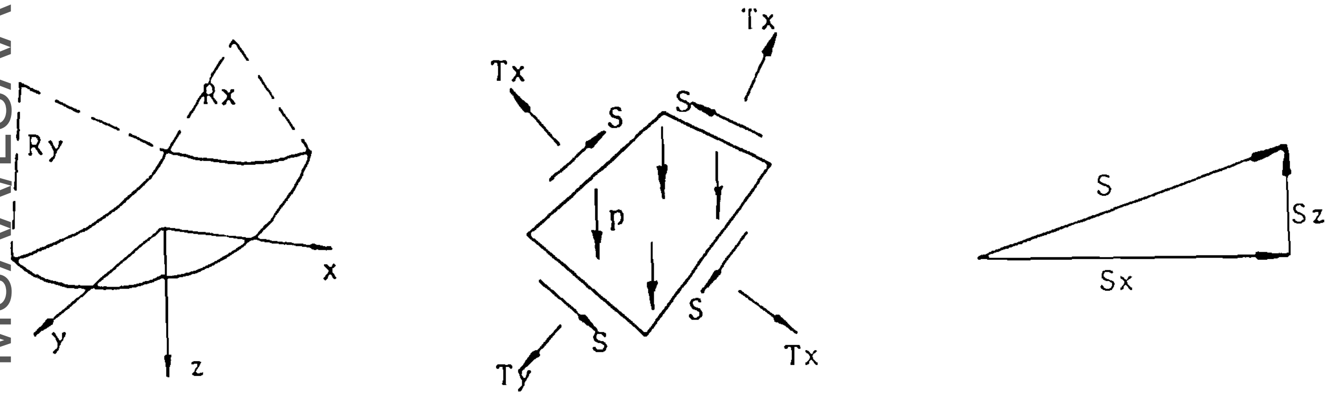


Fig (4)

$$P = \frac{Tx}{Rx} + \frac{Ty}{Ry} + \frac{2S}{Rxy}$$

$$P = \frac{T_1}{R_1} + \frac{T_2}{R_2}$$

$$\text{if } P=0 \text{ then } \frac{T_1}{R_1} = \frac{-T_2}{R_2} \text{ or } \frac{T_1}{T_2} = \frac{-R_1}{R_2}$$

Fig (5)

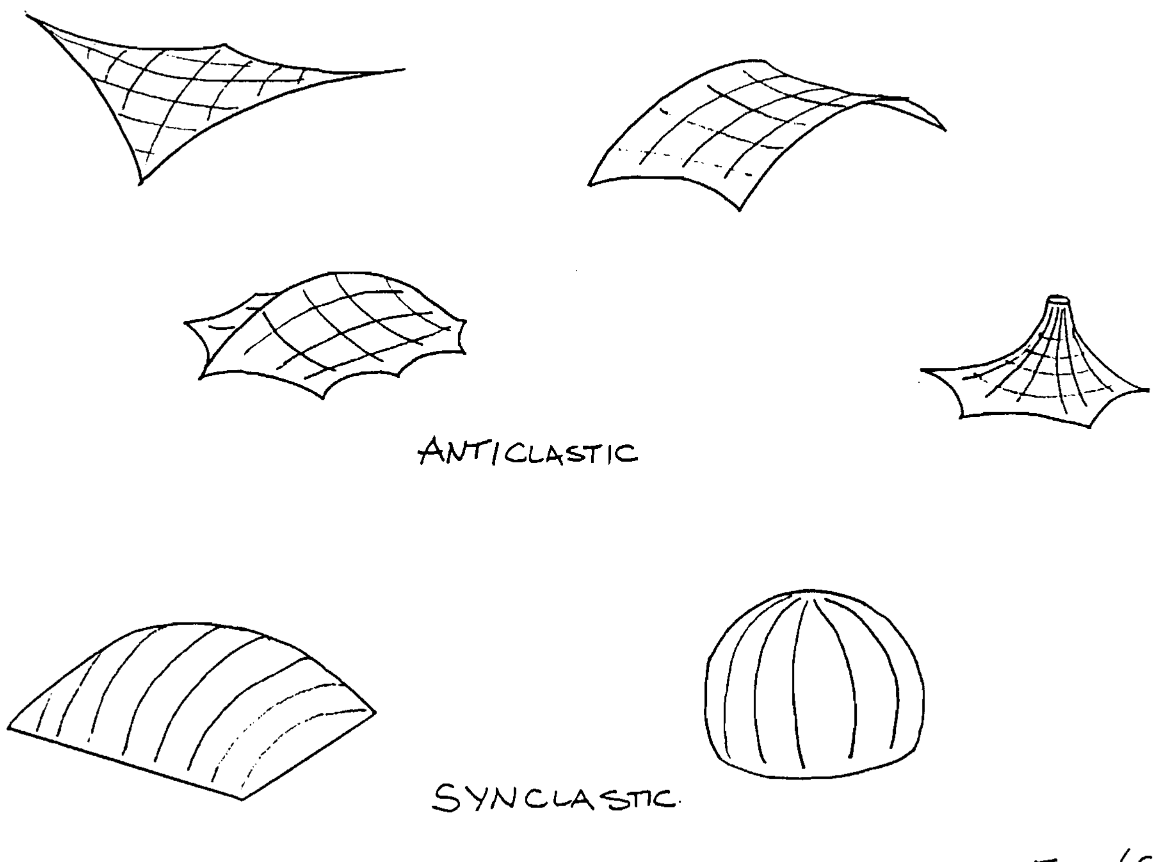


FIG (6)

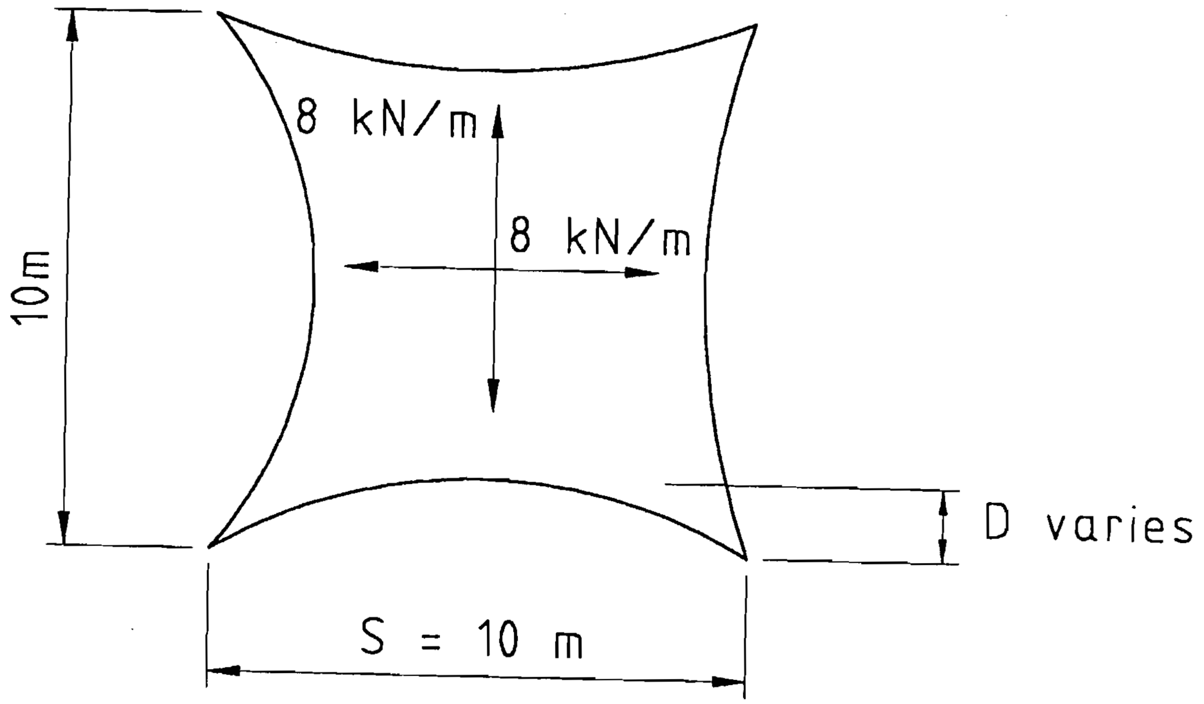

10m
8 kN/m
8 kN/m
D varies
S = 10 m

Fig (7)

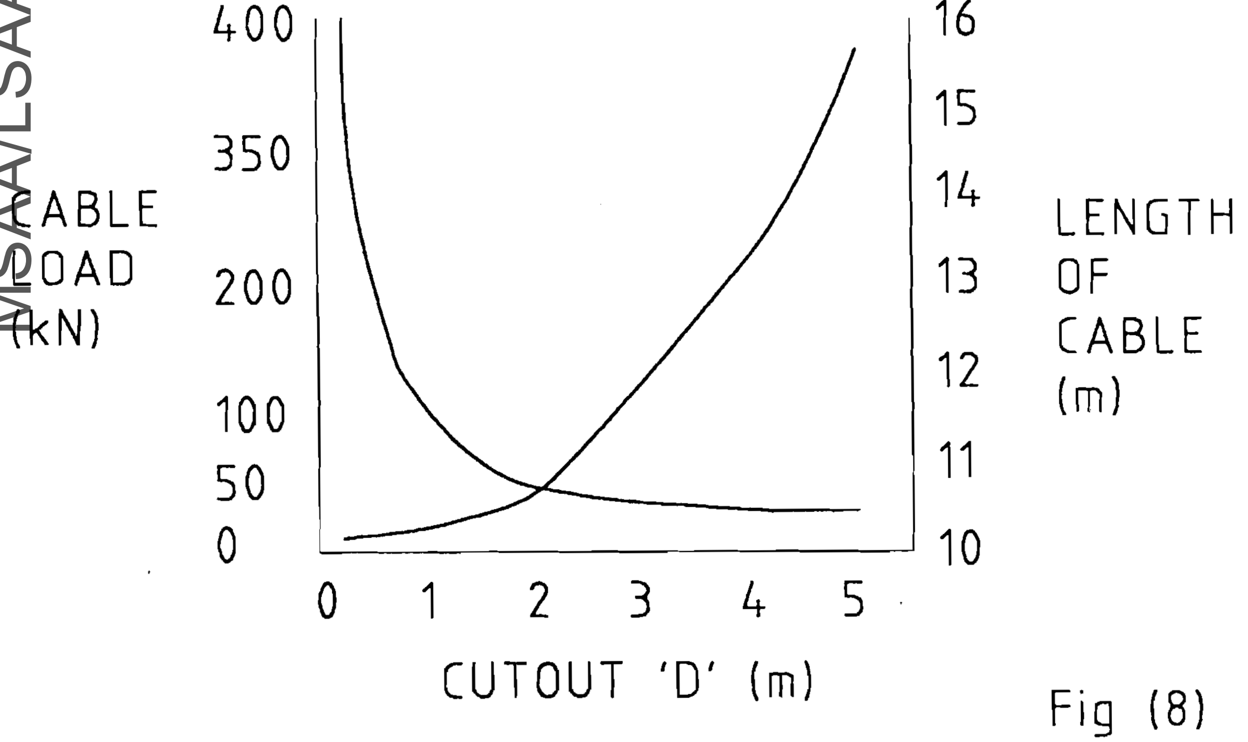

400
350
200
100
50
0
CABLE
LOAD
(kN)
16
15
14
13
12
11
10
LENGTH
OF
CABLE
(m)
0
1
2
3
4
5
CUTOUT 'D' (m)

Fig (8)

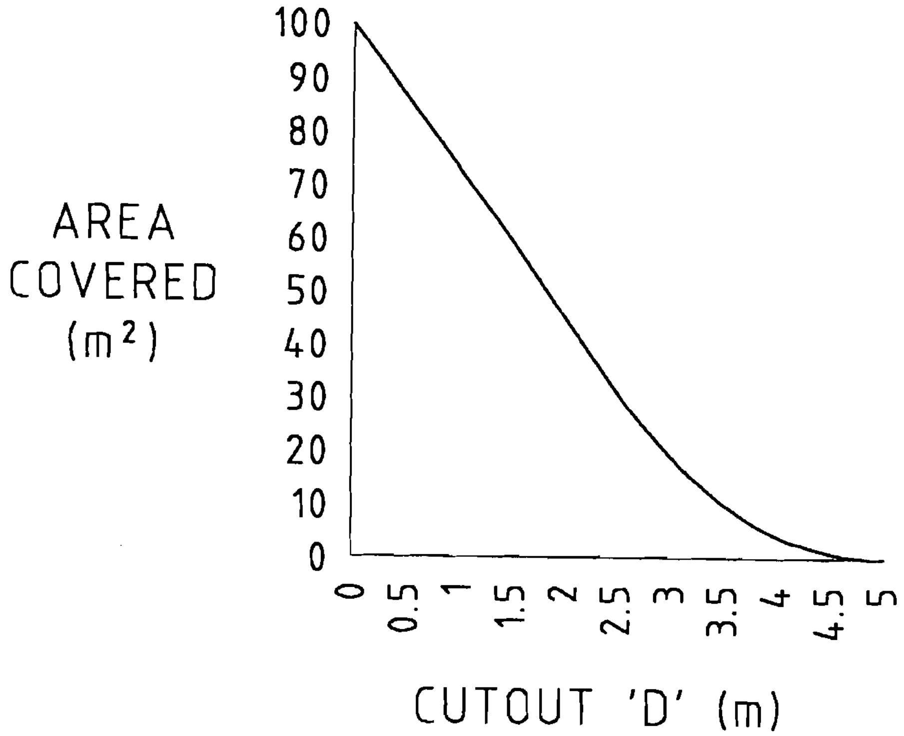


Fig (9)

SUMMARY OF LOADS

Self Weight	0.02 kPa	Vertical
Prestress	1-2.5 kN/m	In Plane of fabric
Live Loads	0.25 kPa	Vertical
Wind Loads	0.4 - 2.2 kPa	Normal to fabric
Earthquake Loads	% of Self-weight	Horizontal
Snow Loads	0.5 - 3 kPa	Vertical plus drifts

Fig (10)